JEE MAINS-2016

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particular on this page of the Test Booklet with only Blue / Black Ball Point Pen provided by the Board.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Mathematics, Physics and Chemistry having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
6. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side–1 and Side–2 of the Answer Sheet use only Blue/Black Ball Point Pen provided by the Board.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is F. Make sure that the CODE printed on Side–2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray mark on the Answer Sheet.

PART-A-MATHEMATICS

1. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is

(1*) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Sol. $\frac{2+3i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}=\frac{2-6\sin^2\theta}{1+4\sin^2\theta}+i\frac{7\sin\theta}{1+4\sin^2\theta}$

For purely imaginary,

$2 - 6\sin^2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

2. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for

(1*) exactly three values of λ. (2) infinitely many values of λ.

(3) exactly one value of λ. (4) exactly two values of λ.

Sol. For non-trivial solution

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$

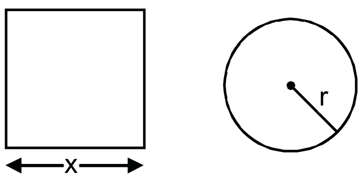
$\lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$

$\lambda^3 - \lambda = 0 \Rightarrow \lambda = 0, \pm 1$.

3. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(1) $2x = r$ (2) $2x = (\pi + 4)r$ (3) $(4 - \pi)x = \pi r$ (4*) $x = 2r$

Sol.



$4x + 2\pi r = 2 \Rightarrow 2x + \pi r = 1$

$A = x^2 + \pi r^2 = x^2 + \pi\left(\frac{1-2x}{\pi}\right)^2 = \frac{1}{\pi}\left(\pi x^2 + (4x^2 - 4x + 1)\right) = \frac{1}{\pi}\left((\pi + 4)x^2 - 4x + 1\right)$

$$\frac{dA}{dx} = \frac{1}{\pi}(2x(\pi + 4) - 4) = 0$$

$$x = \frac{2}{\pi + 4} \qquad \text{(point of minimum)}$$

$$2x + \pi r = 1.$$

$$\frac{4}{\pi+4} + \pi r = 1 \Rightarrow \qquad \pi r = 1 - \frac{4}{\pi+4} = \frac{\pi}{\pi+4}$$

$$\therefore\ r = \frac{1}{\pi+4} \Rightarrow \qquad x = 2r.$$

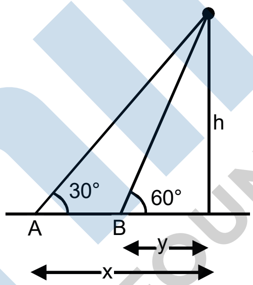
4. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is

(1*) 5 (2) 6 (3) 10 (4) 20

Sol. $h = x\tan 30° = y \tan 60°$

$\Rightarrow x = 3y$

$\because$ speed is constant



$$\therefore \frac{x-y}{10} = \frac{y}{t}$$

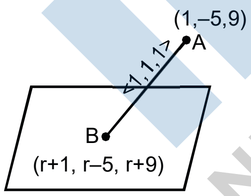
$\Rightarrow t = 5.$

5. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

(1*) E_1, E_2 and E_3 are independent. (2) E_1 and E_2 are independent.

(3) E_2 and E_3 are independent. (4) E_1 and E_3 are independent.

Sol.



E_1 : {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)}

E_2 : {(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)}

E_3 : {(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6), (2, 1), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)}

$$P(E_1) = \frac{6}{36} = \frac{1}{6}; P(E_2) = \frac{6}{36} = \frac{1}{6}$$

$$P(E_3) = \frac{18}{36} = \frac{1}{2}$$

$P(E_1 \cap E_2) = \frac{1}{36} = P(E_1)\,P(E_2)$

$\Rightarrow E_1$ & E_2 are independent

$P(E_2 \cap E_3) = \frac{3}{36} = \frac{1}{12} = P(E_1)\,P(E_3)$

$\Rightarrow E_2$ & E_3 are independent

$P(E_1 \cap E_3) = \frac{3}{36} = \frac{1}{12} = P(E_1)\,P(E_3)$

$\Rightarrow E_1$ & E_3 are independent

$P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1)\,P(E_2)\,P(E_3)$.

$\Rightarrow E_1, E_2$ & E_3 are not independent.

6. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2 - 23a + 44 = 0$
(2) $3a^2 - 26a + 55 = 0$
(3*) $3a^2 - 32a + 84 = 0$
(4) $3a^2 - 34a + 91 = 0$

Sol. $\bar{x} = \frac{2+3+a+11}{4} = 4 + \frac{a}{4} \sum (x_i - \bar{x})^2$

$= \left(2+\frac{a}{4}\right)^2 + \left(1+\frac{a}{4}\right)^2 + \left(4-\frac{3a}{4}\right)^2 + \left(7-\frac{a}{4}\right)^2$

$= 70 + \frac{12a^2}{16} - 8a$

$\because \sigma = 3.5 = \frac{7}{2}$

$\Rightarrow \quad \sigma^2 = \frac{7}{2} = \text{var}(x) = \frac{70 + \frac{12a^2}{16} - 8a}{4} \quad \Rightarrow \quad 49 = 70 + \frac{8a^2}{4} - 8a$

$\Rightarrow \quad \frac{3a^2}{4} - 8a + 21 = 0 \quad \Rightarrow \quad 3a^2 - 32a + 84 = 0.$

7. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(1) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

(2) g is not differentiable at x = 0

(3*) $g'(0) = \cos(\log 2)$

(4) $g'(0) = -\cos(\log 2)$

Sol. In the neighbourhood of x = 0

$f(x) = \log 2 - \sin x$

$g(x) = f(f(x)) = \log 2 - \sin(\log 2 - \sin x)$

$\therefore\ g'(x) = -\cos(\log 2 - \sin x)\,(-\cos x)$

$\Rightarrow \qquad g'(0) = \cos(\log 2)$

Aliter: $g'(0^+) = \lim_{h\to 0} \frac{f(f(0+h)) - f(f(0))}{h} = \lim_{h\to 0} \frac{f(\log 2 - \sin h) - |\log 2 - \sin(\log 2)|}{h}$

$= \lim_{h\to 0} \frac{|\log 2 - \sin(\log 2 - \sin h)| - |\log 2 - \sin(\log 2)|}{h} = \lim_{h\to 0} \frac{\sin(\log 2) - \sin(\log 2 - \sin h)}{h} =$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{2\log 2 - \sin h}{2}\right)\sin\left(\frac{\sin h}{2}\right)}{h} = \cos(\log 2)$

$g'(0^-) = \lim_{h\to 0} \frac{f(f(0-h)) - f(f(0))}{-h} = \lim_{h\to 0} \frac{f(\log 2 + \sin h) - f(\log 2)}{-h}$

$= \lim_{h\to 0} \frac{|\log 2 - \sin(\log 2 + \sin h)| - |\log 2 - \sin(\log 2)|}{-h}$

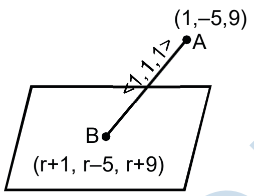
$= \lim_{h\to 0} \frac{\sin(\log 2 + \sin h) - \sin(\log 2)}{h}$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{2\log 2 + \sin h}{2}\right)\sin\left(\frac{\sin h}{2}\right)}{h} = \cos(\log 2)$.

8. The distance of the point (1, – 5, 9) from the plane x – y + z = 5 measured along the line x = y = z is

(1) $\frac{20}{3}$ (2) $3\sqrt{10}$ (3*) $10\sqrt{3}$ (4) $\frac{10}{\sqrt{3}}$

Sol.



Equation of straight line parallel to x = y = z and passing through (1, –5, 9) is

$\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = r$

x = r + 1, y = r – 5, z = r + 9

x – y + z = 5

r + 1 – (r – 5) + r + 9 = 5

r = – 10.

$B \equiv (-9, -15, -1)$

$\therefore$ AB = $\sqrt{100+100+100} = 10\sqrt{3}$.

9. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(1) $\sqrt{3}$ (2) $\frac{4}{3}$ (3) $\frac{4}{\sqrt{3}}$ (4*) $\frac{2}{\sqrt{3}}$

Sol. 2a = 8 $\Rightarrow$ a = 4

$2b = \frac{1}{2}(2ae) \Rightarrow 2b = 4e \Rightarrow b = 2e$

$e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{4e^2}{16}$

$e^2 = 1 + \frac{e^2}{4} \Rightarrow \frac{3e^2}{4} = 1 \Rightarrow e = \frac{2}{\sqrt{3}}$.

10. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(1) $x^2 + y^2 - 4x + 9y + 18 = 0$ (2*) $x^2 + y^2 - 4x + 8y + 12 = 0$

(3) $x^2 + y^2 - x + 4y - 12 = 0$ (4) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

Sol. Normal to the parabola $y^2 = 8x$

$y = mx - 2am - am^3$

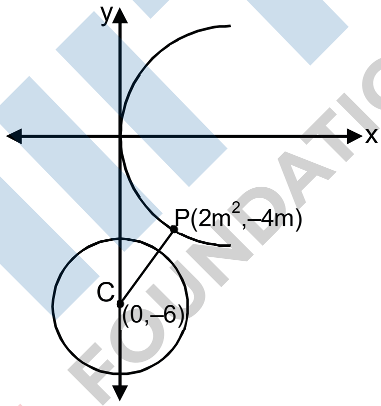
which passes through (0, –6)

$-6 = 0 - 2am - am^3$

$-6 = -4m - 2m^3$

$m^3 + 2m - 3 = 0$

$m = 1$



$\therefore$ P $\in$ (2, – 4)

Radius of the circle, $r = CP = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$.

Equation of the required circle is

$(x - 2)^2 + (y + 4)^2 = 8$

$x^2 + y^2 - 4x + 8y + 12 = 0$.

11. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj. A = A A^T, then 5a + b is equal to

(1) 13 (2) – 1 (3*) 5 (4) 4

Sol. A adj. A = A A^T

$|A| I_n = A A^T$

$(10a + 3b) I_n = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 9 + 4 \end{bmatrix}$

$10a + 3b = 13$, $10a + 3b = 25a^2 + b^2$,

$15a - 2b = 0 \Rightarrow a = \frac{2b}{15}$

$10 \times \frac{2b}{15} + 3b = 13$

$4b + 9b = 39 \Rightarrow b = 3,\ a = \frac{2}{5}$

$5a + b = 5.$

12. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right),\ x \in \left(0, \frac{\pi}{2}\right)$.

A normal to y = f(x) at $x = \frac{\pi}{6}$ also passes through the point

(1) $\left(\frac{\pi}{4}, 0\right)$ (2) (0, 0) (3*) $\left(0, \frac{2\pi}{3}\right)$ (4) $\left(\frac{\pi}{6}, 0\right)$

Sol. $f(x) = \tan^{-1}\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$

$f'(x) = \frac{1}{2}$

$f\left(\frac{\pi}{6}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Equation of normal at $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ is

$y - \frac{\pi}{3} = -2\left(x - \frac{\pi}{6}\right) \Rightarrow 2x + y - \frac{2\pi}{3} = 0$

which passes through $\left(0, \frac{2\pi}{3}\right)$.

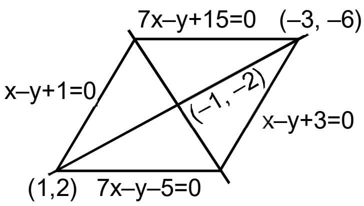
13. Two sides of a rhombus are along the lines, x – y + 1 = 0 and 7x – y – 5 = 0. If its diagonals intersect at (– 1, – 2), then which one of the following is a vertex of this rhombus?

(1) $\left(\frac{-10}{3}, \frac{-7}{3}\right)$ (2) (– 3, – 9) (3) (– 3, – 8) (4*) $\left(\frac{1}{3}, \frac{-8}{3}\right)$

Sol. $7x - y + \lambda = 0$

$-21 + 6 + \lambda = 0$

$\lambda = 15$



$7x - y + 15 = 0$

$x - y + k = 0$

$-3 + 6 + k = 0 \Rightarrow k = -3$

$x - y - 3 = 0$

$x - y - 3 = 0$

$7x - y - 5 = 0$

$-6x + 2 = 0$

$x = \frac{1}{3},\ y = \frac{-5}{3}$

$x - y + 1 = 0$

$7x - y + 15 = 0$

$-6x - 14 = 0$

$x = \frac{-7}{3}$

$y = \frac{-7}{3} + 1 = \frac{-4}{3}.$

Aliter: Equation of angle bisector between

$x - y + 1 = 0$ and $7x - y - 5 = 0$

$$\frac{x - y + 1}{\sqrt{2}} = \pm \frac{7x - y - 5}{5\sqrt{2}}$$

$\Rightarrow 5x - 5y + 5 = \pm (7x - y - 5)$

taking positive sign, $x + 2y - 5 = 0$

taking negative sign, $2x - y = 0$

$2x - y = 0$ which passes through $(-1, -2)$

Another diagonal is

$x + 2y + \lambda = 0 \Rightarrow -1 - 4 + \lambda = 0 \Rightarrow \lambda = 5$

$\therefore\ x + 2y + 5 = 0$

Now, solving $x + 2y + 5 = 0$ and $7x - y - 5 = 0$

we get $\left(\frac{1}{3}, \frac{-8}{3}\right)$

14. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation $y(1 + xy)\,dx = x\,dy$, then $f\left(\frac{-1}{2}\right)$ is equal to

(1*) $\frac{4}{5}$ (2) $\frac{-2}{5}$ (3) $\frac{-4}{5}$ (4) $\frac{2}{5}$

Sol. $y(1 + xy)\,dx = x\,dy$

$ydx - xdy = -xy^2\,dx$

$$\frac{ydx - xdy}{y^2} = -x\,dx$$

$$d\left(\frac{x}{y}\right) = -x\,dx$$

$$\frac{x}{y} = \frac{-x^2}{2} + C$$

$-1 = \frac{-1}{2} + C \Rightarrow C = \frac{-1}{2}$

$\frac{x}{y} = \frac{-x^2}{2} - \frac{1}{2} \Rightarrow \frac{-2x}{y} = x^2 + 1 \Rightarrow y = \frac{-2x}{x^2+1}$

$f\left(\frac{-1}{2}\right) = \frac{-2\left(\frac{-1}{2}\right)}{\frac{1}{4}+1} = \frac{4}{3}.$

15. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is

(1*) 58^{th} (2) 46^{th} (3) 59^{th} (4) 52^{nd}

Sol. SMALL

A _ _ _ _ = $\frac{4!}{2!}$ = 12

L _ _ _ _ = 4! = 24

M _ _ _ _ = $\frac{4!}{2!}$ =12

S A _ _ _ = $\frac{3!}{2!}$ = 3

S L _ _ _ = 3! = 6

S M A L L = 1

―――

58^{th}.

Hence, rank of word SMALL is 58.

16. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is

(1) $\frac{7}{4}$ (2) $\frac{8}{5}$ (3*) $\frac{4}{3}$ (4) 1

Sol. Let a, ar, ar^2 be the three consecutive terms of GP and d be the common difference of AP

$\therefore$ ar – a = 3d

and ar^2 – ar = 4d

$\Rightarrow \frac{ar-a}{3} = \frac{ar^2-ar}{4}$

$\frac{(r-1)}{3} = \frac{r(r-1)}{4} \Rightarrow r = \frac{4}{3}.$

17. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, x ≠ 0, is 28, then the sum of the coefficients of all the terms in this expansion, is

(1*) 729 (2) 64 (3) 2187 (4) 243

Sol. Put x = 1

3^n = sum of coefficient

Number of terms in $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$ is 28.]

2n + 1 = 28 (not possible).

Incorrect Solution:

Number of terms $^{n+2}C_2 = 28$

$n + 2 = 6 \Rightarrow n = 6$

Put x = 1.

Sum: $(1 - 2 + 4)^6 = 3^6 = 729$.

18. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots$, is $\frac{16}{5}$m, then m is equal to

(1) 99 (2) 102 (3*) 101 (4) 100

Sol. $T_n = \left(\frac{4n+4}{5}\right)^2 = \frac{16}{25}(n+1)^2$

$S_{10} = \frac{16}{25}(2^2 + 3^2 + \ldots\ldots. + 11^2)$

$= \frac{16}{25}\left(\frac{11\cdot 12\cdot 23}{6}-1\right) = \frac{16}{25}(505)$

$= \frac{16}{5}\cdot(101)$.

19. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to

(1*) 2 (2) 26 (3) 18 (4) 5

Sol. $L : \frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$

$P : \ell x + my - z = 9$, the

point (3, – 2, – 4) lies on the plane

$3\ell - 2m + 4 = 9$

$\Rightarrow 3\ell - 2m = 5$ ....(1)

$\bar{n}\cdot\bar{b} = 2\ell - m - 3 = 0$ ....(2)

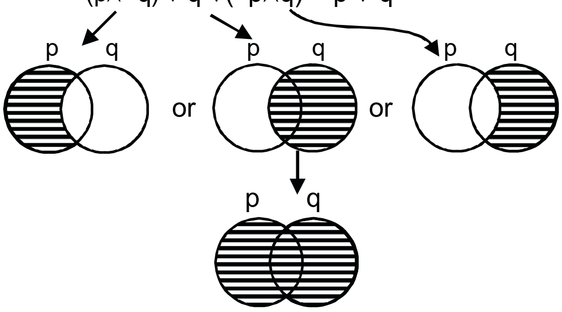
Solving (1) & (2), $\ell = 1$ and $m = -1$

$\therefore \ell^2 + m^2 = 2$.

20. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to

(1) $p \vee \sim q$ (2) $\sim p \wedge q$ (3) $p \wedge q$ (4*) $p \vee q$

Sol. $(p \wedge \sim q) \vee q \vee (\sim p \wedge q) = p \vee q$



21. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3} dx$ is equal to

(1) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (2) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(3*) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (4) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.

Sol. $\int \frac{(2x^{12}+5x^9)}{x^{15}\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)} dx = \int \frac{\frac{2}{x^3}+\frac{5}{x^6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3} dx$

Put 1 + $\frac{1}{x^2}+\frac{1}{x^5}$ = t

$-\left(\frac{2}{x^3}+\frac{5}{x^6}\right)$ dx = dt

$= \int \frac{-dt}{t^3} = \frac{1}{2t^2}+C = \frac{x^{10}}{2\,(x^5+x^3+1)^2}+C$.

22. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then radius of S is

(1) 10 (2) $5\sqrt{2}$ (3*) $5\sqrt{3}$ (4) 5

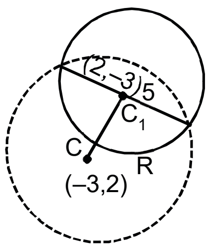
Sol. C_1 (2, – 3)

$r_1 = \sqrt{4+9+12} = 5$

$CC_1 = \sqrt{5^2+5^2} = 5\sqrt{2}$

$R^2 = 25 + 50 = 75$

$R = 5\sqrt{3}$.



23. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)\,.......\,3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

(1) 3log 3 – 2 (2) $\frac{18}{e^4}$ (3*) $\frac{27}{e^2}$ (4) $\frac{9}{e^2}$

Sol. $\ell = \lim_{x\to\infty}\left(\frac{(n+1)(n+2)\ldots\ldots(n+2n)}{n^{2n}}\right)^{\frac{1}{n}}$

$\ell = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\ldots\left(1+\frac{2n}{n}\right)\right)^{\frac{1}{n}}$

$\ln \ell = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right) = \int_0^2 \ln(1+x)\,dx = x\cdot\ln(1+x)\big|_0^2 - \int_0^2\frac{x+1-1}{x+1}dx$

$= 2\,\ell n\,3 - (\,x - \ln(1+x)\,)\big|_0^2$

$= 2\,\ell n\,3 - (2 - \ln 3)$

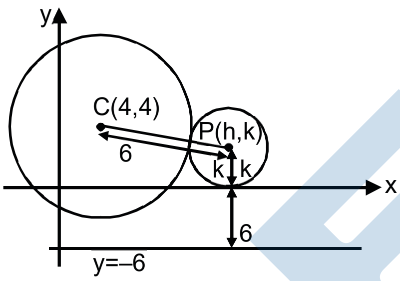
$= 3\,\ell n\,3 - 2$

$\Rightarrow \ell = \frac{27}{e^2}$.

24. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on

(1*) a parabola.
(2) a circle.
(3) an ellipse which is not a circle.
(4) a hyperbola.

Sol.



Clearly, locus will be a parabola as P moves such that it is always at equal distance from a fixed point (4, 4) & fixed line y = – 6.

Answer is 1.

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. if $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1*) $\frac{5\pi}{6}$
(2) $\frac{3\pi}{4}$
(3) $\frac{\pi}{2}$
(4) $\frac{2\pi}{3}$

Sol. $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow\ (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

$\therefore\ \vec{a}\cdot\vec{b} = \frac{-\sqrt{3}}{2} \Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{5\pi}{6}$.

26. Let p = $\lim_{x \to 0^+}(1+\tan\sqrt[2]{x})^{\frac{1}{2x}}$ then log p is equal to

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4*) $\frac{1}{2}$

Sol. $p = \lim_{x \to 0^+}(1+\tan\sqrt[2]{x})^{\frac{1}{2x}} = e^{\lim_{x \to 0^+}\frac{1}{2x}(\tan\sqrt[2]{x})} = e^{\frac{1}{2}}$

$\therefore$ log p = $\frac{1}{2}$.

27. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

cos x + cos 2x + cos 3x + cos 4x = 0, is

(1) 9 (2) 3 (3) 5 (4*) 7

Sol. cos x + cos 2x + cos 3x + cos 4x = 0

$\Rightarrow \; 2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \; 2\cos\frac{5x}{2}\left(2\cos x\cos\frac{x}{2}\right) = 0$

$\Rightarrow \cos\frac{x}{2} = 0$ OR cos x = 0 OR $\cos\frac{5x}{2} = 0$

$\Rightarrow \; \frac{x}{2} = (2n+1)\frac{\pi}{2}$ OR $x = (2m+1)\frac{\pi}{2}$ OR $\frac{5x}{2} = (2k+1)\frac{\pi}{2} \Rightarrow x = (2k+1)\frac{\pi}{5}$

$\because 0 \le x < 2\pi$

$\therefore x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

$\therefore$ Number of solutions = 7.

28. The sum of all real values of x satisfying the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

(1) 5 (2*) 3 (3) – 4 (4) 6

Sol. $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$

(i) If $x^2 - 5x + 5 = 1 \Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow$ x = 1 or 4

(ii) If $x^2 + 4x - 60 = 0 \Rightarrow (x + 10)(x - 6) = 0$

$\Rightarrow$ x = – 10 or 6

(iii)If $x^2 - 5x + 5 = -1$

and $x^2 + 4x - 60$ = even integer

$\Rightarrow \quad x^2 - 5x + 6 = 0$

$\Rightarrow \quad x = 2$ or $x = 3$

For x = 2,

$x^2 + 4x - 60 = 4 + 8 - 60 = -48$ = even

For x = 3,

$x^2 + 4x - 60 = 9 + 12 - 60 = -39$ = odd

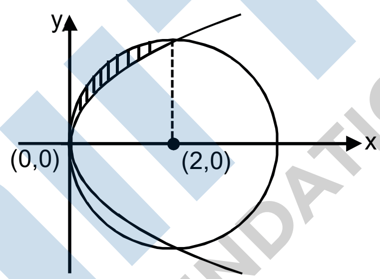
$\therefore$ x = 2 is also a solution.

Sum of values of x = 3.

29. The area (in sq. units) of the region $\{(x, y): y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is

(1) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (2) $\pi - \frac{4}{3}$ (3*) $\pi - \frac{8}{3}$ (4) $\pi - \frac{4\sqrt{2}}{3}$

Sol. $x^2 + y^2 \leq 4x \Rightarrow (x-2)^2 + y^2 \leq 4$



$y^2 \geq 2x$

Point of intersection = $x^2 + 2x = 4x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0$ or $x = 2$

Area bounded

$= \left(\frac{1}{4}\right)^{th}$ area of circle $- \int_0^2 \sqrt{2x}\, dx$

$= \frac{1}{4} \times \pi \cdot 2^2 - \left(\frac{\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 = \pi - \frac{8}{3}.$

30. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S

(1) contains more than two elements. (2) is an empty set.

(3) contains exactly one element. (4*) contains exactly two elements.

Sol. $\because \quad f(x) + 2f\left(\frac{1}{x}\right) = 3x \quad \ldots(1)$

replace x by $\frac{1}{x}$, we get

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \quad \ldots(2) \times 2$

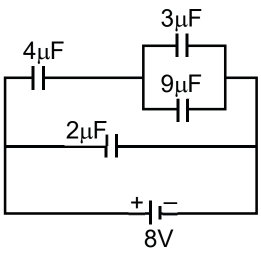
$-3f(x) = 3x - \frac{6}{x} \Rightarrow f(x) = \frac{2}{x} - x = \frac{2 - x^2}{x}$

$\because \quad f(-x) = f(x) \Rightarrow \frac{2 - x^2}{-x} = 2 - x^2 = 0$

$\Rightarrow x = \pm\sqrt{2}$.

PART-B-PHYSICS

31. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 9μF capacitors), at a point distant 30 m from it, would equal :



(1) 480 N/C (2) 240 N/C (3) 360 N/C (4*) 420 N/C

Sol. $Q = Q_4 + Q_9$

$= 24\ \mu C + 18\ \mu C = 42\ \mu C$

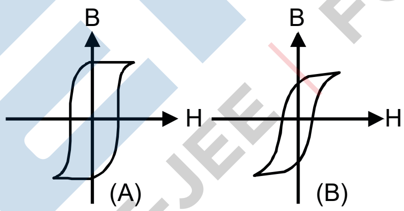
$E = \dfrac{kQ}{r^2} = 420$ N/C

32. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears:

(1*) 20 times nearer. (2) 10 times taller. (3) 10 times nearer. (4) 20 times taller.

Sol. Apply the concept of visual angle

33. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(1*) B for electromagnets and transformers.

(2) A for electric generators and transformers.

(3) A for electromagnets and B for electric generators.

(4) A for transformers and B for electric generators .

Sol. Information based

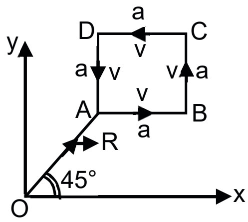
34. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be' :

(1*) 5 : 4 (2) 1 : 16 (3) 4 : 1 (4) 1 : 4

Sol. $N_{Decayed} = No\left(1 - e^{-\frac{t}{\tau_{1/2}}}\right)$

$$\frac{N_A}{N_B} = \frac{5}{4}$$

35. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure:



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin ?

(1*) $\vec{L} = \frac{m\upsilon}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from D to A.

(2) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}} R\,\hat{k}$ when the particle is moving from A to B.

(3*) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(4) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

Sol. Angular momentum = $\vec{r} \times \vec{p}$

36. Choose the correct statement:

(1) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(2*) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal

(3) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(4) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

Sol. The instantaneous value of the career wave's amplitude changes in accordance with the amplitude and frequency variations of the modulating single.

37. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index ?

(1) 1.8 (2*) 1.5 (3) 1.6 (4) 1.7

Sol. $\delta = 1 + e - A \Rightarrow A = 74°$

$$\mu = \frac{\sin\left(\frac{a+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{5}{3}\sin\left(37° + \frac{\delta_{min}}{2}\right)$$

μ_{max} can be $\frac{5}{3}$, so μ will be less than $\frac{5}{3}$

Since δ_{min} will be less then 40°. So

$$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \Rightarrow \mu < 1.446$$

So the nearest possible value of μ should be 1.5

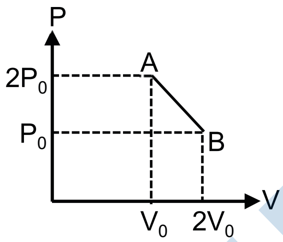
38. 'n' moles of an ideal gas undergoes a process A→B as shown in the figure. The maximum temperature of the gas during the process will be :

(1) $\frac{9P_0V_0}{nR}$

(2*) $\frac{9P_0V_0}{4nR}$

(3) $\frac{3P_0V_0}{2nR}$

(4) $\frac{9P_0V_0}{2nR}$



Sol. $P = -\frac{P_0}{V_0}V + 3P_0 = \frac{nRT}{V}$

$$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + 3P_0V\right]$$

$$\frac{dT}{dV} = 0 \qquad V = \frac{3V_0}{2}$$

$$T = \frac{9P_0V_0}{4nR}$$

39. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and. square respectively, then the ratio $\frac{B_A}{B_B}$ is :

(1*) $\frac{\pi^2}{8\sqrt{2}}$ (2) $\frac{\pi^2}{8}$ (3) $\frac{\pi^2}{16\sqrt{2}}$ (4) $\frac{\pi^2}{16}$

Sol. $B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I(2\pi)}{2\ell}$ $\quad$ [$l = 2\pi R$]

$B_B = 4\left[\frac{\mu_0 I\sqrt{2}}{4\pi\frac{a}{2}}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi a} = \frac{8\sqrt{2}\mu_0 I}{\pi l}$ $\quad$ [$\ell = 4a$]

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

40. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line ?

(1) 0.50 mm (2) 0.75 mm (3*) 0.80 mm (4) 0.70 mm

Sol. Reading $= 0.5 + 25\left(\frac{0.5}{50}\right) + 5\left(\frac{0.5}{50}\right) = 0.8$ mm

41. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is:

(1*) $\alpha = \frac{\beta^2}{1+\beta^2}$ (2) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (3*) $\alpha = \frac{\beta}{1-\beta}$ (4) $\alpha = \frac{\beta}{1+\beta}$

Sol. $I_\varepsilon = I_B + I_C$

$$\frac{I_\varepsilon}{I_C} = \frac{I_B}{I_C} + 1$$

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\alpha = \frac{\beta}{1+\beta}$$

42. The box of a pin hole camera of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when:

(1) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ (2) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(3) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$ (4*) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

Sol. $a \sin\theta = \lambda$ $\Rightarrow \sin\theta = \frac{\lambda}{a}$

speed = b = a + L tan θ

$= a + \frac{\lambda L}{a}$ [$\tan\theta = \simeq \sin\theta - \frac{\lambda}{a}$]

$$\frac{db}{da} = 0 \Rightarrow 1 - \frac{\lambda L}{a^2} = 0$$

$\Rightarrow$ a = $\sqrt{\lambda L}$

b_{min} = $\sqrt{\lambda L} + \sqrt{\lambda L} = \sqrt{4\lambda L}$

43. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:

(1*) 12.89×10^{-3} Kg (2) 2.45×10^{-3} kg (3) 6.45×10^{-3} kg (4) 9.89×10^{-3} kg

Sol. $m = \dfrac{(10 \times 9.8 \times 1) \times 1000}{3.8 \times 10^7}$ kg ×5

$= 12.89 \times 10^{-3}$ kg

44. Arrange the following electromagnetic radiations per quantum in the order of increasing energy :

A: Blue light B: Yellow light

C : X-ray D : Radiowave.

(1) B, A, D, C (2*) D, B, A, C (3) A, B, D, C (4) C, A, B, D

Sol. $E = \dfrac{hc}{\lambda}$

so the order is D, B, A, C

45. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :

(1) $n = \dfrac{C - C_V}{C - C_P}$ (2) $n = \dfrac{C_P}{C_V}$ (3*) $n = \dfrac{C - C_P}{C - C_V}$ (4) $n = \dfrac{C_P - C}{C - C_V}$

Sol. $C = C_V + \dfrac{R}{1-n}$

$\Rightarrow 1 - n = \dfrac{R}{C - C_V}$

$\Rightarrow n = 1 - \dfrac{R}{C - C_V} = \dfrac{C - C_V - R}{C - C_V}$

$\therefore\ n = \dfrac{C - C_P}{C - C_V}$

46. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere.)

(1*) $\sqrt{gR}\left(\sqrt{2} - 1\right)$ (2) $\sqrt{2gR}$ (3) $\sqrt{gR}$ (4) $\sqrt{gR/2}$

Sol. $\dfrac{1}{2}mv^2 - \dfrac{Gmn}{R+h} = 0$

$\Rightarrow \upsilon = \sqrt{\dfrac{2GM}{R+h}}$

$$\Delta\upsilon = \sqrt{\frac{2GM}{R+h}} - \sqrt{\frac{GM}{R+h}}$$

$$= \left(\sqrt{2}-1\right)\sqrt{\frac{GM}{R+h}} \approx \left(\sqrt{2}-1\right)\sqrt{gR}$$

47. A galvanometer having a coil resistance of 100Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is :

(1) 3Ω (2*) 0.01Ω (3) 2 Ω (4) 0.1Ω

Sol. $100 \times 10^{-3} \approx 10 \times S$

$S = 10^{-2}\ \Omega = 0.01\ \Omega$

48. Radiation of wavelength λ, is incident on photocell. The fastest emitted electron has speed υ. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be :

(1) $=\upsilon\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (2*) $>\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (3) $<\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (4) $=\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Sol. $\frac{hc}{\lambda} - \varphi = \frac{1}{2}mv^2$

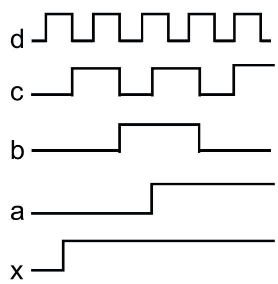
$\frac{4hc}{3\lambda} - \varphi = \frac{1}{2}mv'^2$

$\Rightarrow \frac{hc}{3\lambda} = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}mv'^2 = \frac{1}{2}mv^2 \frac{hc}{2\lambda} > \frac{1}{2}mv^2 + \frac{1}{6}mv^2$

$\Rightarrow v'^2 > \frac{4v^2}{3}$

49. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is :



(1) NAND (2) NOT (3) AND (4*) OR

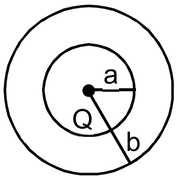
Sol. OR gate $\Rightarrow$ A + B

0 + 1 = 1

1 + 0 = 1

0 + 0 = 0

50. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density ρ = , Where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :



(1) $\frac{2Q}{\pi a^2}$

(2*) $\frac{Q}{2\pi a^2}$

(3) $\frac{Q}{2\pi\left(b^2 - a^2\right)}$

(4) $\frac{2Q}{\pi\left(a^2 - b^2\right)}$

Sol. $\varphi \vec{E}.d\vec{s} = \frac{q_{in}}{q_0}$

$$E\,4\pi r^2 = \frac{\left[Q + \int_a^r 4\pi r^2 dr \frac{A}{r}\right]}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{Q + \int_a^r 4\pi r\, drA}{\varepsilon_0}$$

$$E\,4\pi r^2 = \frac{Q + 2\pi A(r^2 - a^2)}{4\pi r^2 \varepsilon_0}$$

$$= \frac{Q}{4\pi r^2 \varepsilon_0} + \frac{2\pi A r^2}{4\pi r^2 \varepsilon_0} - \frac{2\pi A a^2}{4\pi r^2 \varepsilon_0}$$

$$\frac{Q}{4\pi r^2 \varepsilon_0} = \frac{2\pi A a^2}{4\pi r^2 \varepsilon_0}$$

$$A = \frac{Q}{2\pi a^2}$$

51. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :

(1) 92 ± 3 s (2*) 92 ± 2 s (3) 92 ± 5.0 s (4) 92 ± 1.8 s

Sol. Result can be reported using standard deviation

Standed deviation $\sigma = \sqrt{\frac{r_1^2 + r_2^2 + \dots r_n^2}{n}}$ ...(i)

but strictly

$\sigma = \sqrt{\frac{\delta_1^2 + \delta_2^2 + \dots \delta_n^2}{n-1}}$ ...(ii)

So using equation (ii)

$$\sigma = \sqrt{\frac{(90-92)^2+(91-92)^2+(95-92)^2+(92-92)^2}{4-1}}$$

$$\sigma = \sqrt{\frac{14}{3}} = 2.16$$

So reported result should be using conservative approch

T = (92 ± 3)

If we use equation (i) we get

T = (92 ± 2)

So for the sanctity of science the answer should be (1)

52. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by :

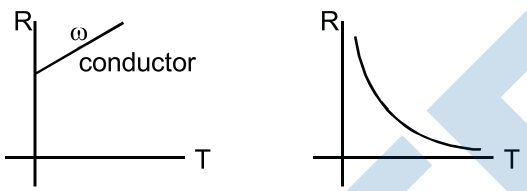
(1) Linear decrease for Cu, linear decrease for Si.

(2) Linear increase for Cu, linear increase for Si.

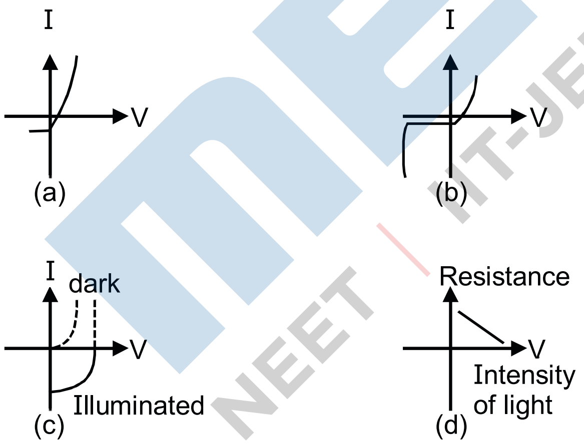
(3) Linear increase for Cu, exponential increase for Si

(4*) Linear increase for Cu, exponential decrease for Si

Sol.



53. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d) :



(1) Zener diode, Solar cell, Simple diode, Light dependent resistance

(2*) Simple diode, Zener diode, Solar cell, Light dependent resistance

(3) Zener diode, Simple diode, Light dependent resistance, Solar cell

(4) Solar cell, Light dependent resistance, Zener diode, Simple diode

Sol. VI characterstics

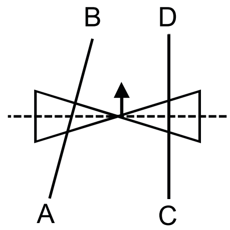
a = normal or simple diode

b = Zener diode (works in R.B sitoation)

c = Solar cell $\Rightarrow$ no biasing require only conducts when illuminated

d = LDR $\rightarrow$ instensity T_1 Resistance

54. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :



(1) turn left and right alternately.
(2*) turn left.
(3) turn right.
(4) go straight.

Sol. From normal reaction of roller, we can conclude it moves towards left.

55. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively :

(1) 55° C; α = 1.85 × 10^{-2} /°C
(2*) 25° C; α = 1.85 × 10^{-5} /°C
(3) 60°C ; α = 1.85 × 10^{-4} /°C
(4) 30° C; α = 1.85 × 10^{-3} /°C

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell} = \frac{1}{2}\alpha\Delta\theta$$

$$12 = \frac{1}{2}\alpha(40-\theta_0)\times 24\times 60\times 60$$

$$4 = \frac{1}{2}\alpha(\theta_0-20)\times 24\times 60\times 60$$

$$3 = \frac{40-\theta_0}{\theta_0-20}$$

$3\theta_0 - \theta_0 = 40 - \theta_0$

$4\theta_0 = 100$

$\theta_0 = 25$

$$12 = \frac{1}{2}\alpha\ 10\times 24\times 60\times 60$$

$$\alpha = \frac{1}{10\times 60\times 60}$$

$$= \frac{1}{36}\times 10^{-3}$$

$\alpha = 1.85 \times 10^{-5}$

56. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is : (take $g = 10\ ms^{-2}$)

(1) $\sqrt{2}s$ (2) $2\pi\sqrt{2}s$ (3) 2s (4*) $2\sqrt{2}s$

Sol. $T = \frac{M}{L}xg$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{xg}$

$a = v\frac{dv}{dx} = \sqrt{xg}\frac{dv}{dx}$

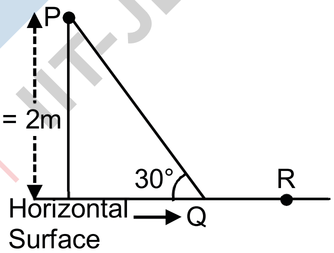
$\sqrt{g}\frac{1}{2\sqrt{x}} = \frac{g}{2}$

$s = \frac{1}{2}at^2$

$t = \frac{2s}{a} = \sqrt{\frac{2 \times 20}{(10/2)}}$

$= 2\sqrt{2}$

57. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ, and the distance x (= QR), are, respectively close to:



(1) 0.29 and 6.5 m (2) 0.2 and 6.5 m (3) 0.2 and 3.5 m (4*) 0.29 and 3.5 m

Sol. μ mg cos30 × 2 cos 30 = μ mg × x ...(i)

$2\sqrt{3} = x$

x = 2 × 1.732 = 3.464 = 3.5 m

mgh – μ mg cos 30 × 2 cos 30

$= \frac{1}{2}mg^2 = \mu\ mg\ x$

mg h – μ mg 2 cot 30 = μ mg 2 $\sqrt{3}$

$1 - \mu\sqrt{3} = \mu\sqrt{3}$

$1 = 2\,\mu\sqrt{3}$

$1 = 2\,\mu\sqrt{3}$

$\mu = \frac{1}{2\sqrt{3}} = \frac{1.732}{2\times 3} = \frac{1.732}{6} = 0.29$

58. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now :

(1*) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f

Sol. $f = \frac{v}{2\ell}$

$f' = \frac{v}{4\times\left(\frac{l}{2}\right)} = \frac{v}{2l} = f$

59. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :

(1*) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

Sol. $v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

$v = \omega\sqrt{\frac{5A^2}{9}}$

$3v = v' = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$

$3\omega\sqrt{\frac{5A^2}{9}} = \omega\sqrt{(A')^2 - \frac{4A^2}{9}}$

$9 \times \frac{5A^2}{9} = (A)^2 - \frac{4A^2}{9}$

$(A')^2 = 5A^2 + \frac{4A^2}{9} = \frac{49A^2}{9}$

$A' = \frac{7A}{3}$

60. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to :

(1*) 0.065 H (2) 80 H (3) 0.08 H (4) 0.044 H

Sol. $R = \frac{V}{I} = \frac{80}{10} = 8$

$$10 = \frac{220}{\sqrt{X_L^2 + 8^2}}$$

$$X_L^2 + 8^2 = 22 \times 22$$

$$X_L^2 = 22^2 - 8^2$$

$$X_L = \sqrt{30 \times 14}$$

$$L = \sqrt{420}$$

$$L = \frac{\sqrt{420}}{2 \times 3.14 \times 50}$$

$$L = \frac{3.263}{50} H$$

$= 0.065$ H

$i_{rms} \times 8 = 80$

$i_{rms} = 10$

PART-C-CHEMISTRY

61. Which one of the following statements about water **is FALSE** ?

(1) Ice formed by heavy water sinks in normal water.

(2) Water is oxidized to oxygen during photosynthesis.

(3) Water can act both as an acid and as a base.

(4*) There is extensive intramolecular hydrogen bonding in the condensed phase.

Sol. Ice shows intermolecular H – bonding.

62. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of :

(1) Iron (2) Fluoride (3) Lead (4*) Nitrate

Sol. Maximum limit of nitrate in potable water is 50 ppm.

63. Galvanization is applying a coating of :

(1*) Zn (2) Pb (3) Cr (4) Cu

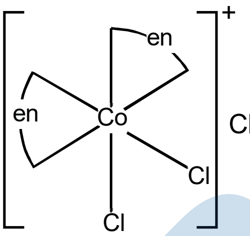
64. Which one of the following complexes shows optical isomerism ?

(1) $[Co(NH_3)_4Cl_2]Cl$ (2) $[Co(NH_3)_3Cl_3]$

(3*) cis$[Co(en)_2Cl_2]Cl$ (4) trans$[Co(en)_2Cl_2]Cl$

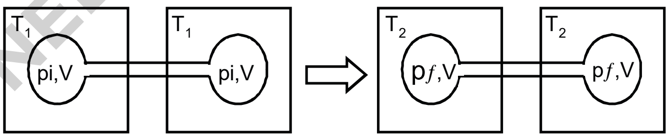
(en = ethylenediamine)

Sol.



Cis $[Co(en_2\ Cl_2)]Cl$
(optically active)

65. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is :



(1) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (2) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (3) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$ (4*) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$

Sol. Intial total moles:

$$n = \frac{p_iV}{RT_1} + \frac{p_iV}{RT_1} = \frac{2p_iV}{RT_1} \quad ....(1)$$

Final total moles:

$$n = \frac{p_f V}{RT_1} + \frac{p_f V}{RT_2} \qquad ....(2)$$

Equating the two:

$$\frac{2p_i}{T_1} = \frac{p_f}{T_1} + \frac{p_f}{T_2}$$

$$p_f = \frac{2p_i}{T_1} \times \frac{T_1 T_2}{(T_1 \times T_2)}$$

$$p_f = 2p_1 \times \frac{T_2}{(T_1 \times T_2)}$$

66. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is :

(1*) –110.5 (2) 110.5 (3) 676 .5 (4) –676.5

Ans. *[–110] (exact answer is –110kJ/mol. closest option is –110.5 kJ/mol)

Sol. $C + O_2 \longrightarrow CO_2$ ....(1)

$$CO + \frac{1}{2}O_2 \longrightarrow CO_2 \qquad ....(2)$$

$\Delta H_1 \Rightarrow$ –393.5 KJ /Mole

$\Delta H_2 \Rightarrow$ –283.5 KJ /mole

$$C + \frac{1}{2}O_2 \longrightarrow CO$$

$\Delta_f H_{(CO,g)} = \Delta H_1 - \Delta H_2$

$\Rightarrow$ – 393.5 –(–283.5)

=–393.5 + 283.5

$\Rightarrow$ –110 KJ /Mole

67. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is :

(1) C_4H_{10} (2) C_3H_6 (3*) C_3H_8 (4) C_4H_8

Sol. None

Volume of $O_2 = 375 \times \frac{20}{100} = 75\,ml$

Volume of gaseous hydrocarbon = 15 ml

$$C_xH_y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O(\ell)$$

$$15\left(X + \frac{y}{4}\right) = 75 \qquad ....(1)$$

$x+\frac{y}{4}=5$

Excess air = 375 – 75 = 300

300 + volume of CO_2 = 300

Volume of CO_2 = 30

15 X = 30

x = 12

(Correct option is not given)

$2+\frac{y}{4}=5$

$\frac{y}{4}=3$

y = 12

C_2H_{12} (not possible)

Hence No answer is correct.

However, considering the equation no. 1 alone, if we put x and y values, 3 and 8 respectively, then equation no. 1 is satisfied and answer will be C_3H_8.

68. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be :

(1) 1.34×10^{-2} mol min^{-1}

(2) 6.93×10^{-2} mol min^{-1}

(3*) 6.93×10^{-4} mol min^{-1} lit^{-1}

(4) 2.66 L min^{-1} at STP

Ans. 3, the rate of formation of O_2 is

6.93×10^{-4} mol min^{-1} lit^{-1} [with correct unit of rate]

Sol. $H_2O_2 \longrightarrow H_2O+\frac{1}{2}O_2$

$2t_{1/2}$ = 50 mins

$t_{1/2}$ = 25 mins

$\therefore k=\frac{0.693}{25}$

$-\frac{d[O_2]}{dt}=\frac{1}{2}\times\frac{0.693}{25}\times 0.05$

$=6.93 \times 10^{-4}$

69. The pair having the same magnetic moment is:

[At. No.: Cr=24, Mn=25, Fe =26, Co =27]

(1) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(2) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(3*) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

Sol. $Cr^{2+} \longrightarrow d^4 \longrightarrow$ No of unpaired electron 4

$Fe^{2+} \longrightarrow d^6 \longrightarrow$ No of unpaired electron 4

Both configuration have 4 unpaired electron 4

70. The species in which the N atom is in a state of sp hybridization is :

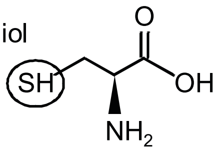
(1) NO_2 (2*) NO_2^+ (3) NO_2^- (4) NO_3^-

Sol. $O = \overset{\oplus}{N} = O$

71. Thiol group is present in :

(1) Methionine (2) Cytosine (3) Cystine (4*) Cysteine

Sol. Thiol



72. The pair in which phosphorous atoms have a formal oxidation state of +3 is :

(1) Pyrophosphorous and pyrophosphoric acids

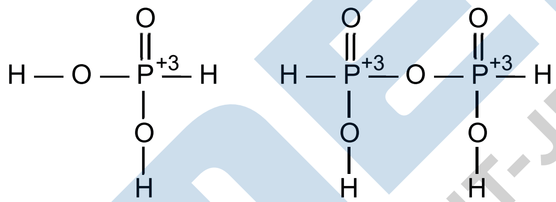
(2*) Orthophosphorous and pyrophosphorous acids

(3) Pyrophosphorous and hypophosphoric acids

(4) Orthophosphorous and hypophosphoric acids

Sol. $H_4P_2O_5 \Rightarrow$ pyrophosphorous (+ 3 state)

$H_3PO_3 \Rightarrow$ orthophosphorous (+ 3 state)



73. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is :

(1*) Distillation under reduced pressure (2) Simple distillation

(3) Fractional distillation (4) Steam distillation

Sol. Glycerol decomposes before its boiling point.

74. Which one of the following ores is best concentrated by froth floatation method?

(1) Malachite (2) Magnetite (3) Siderite (4*) Galena

Sol. Galena (PbS) is a sulphide ore.

75. Which of the following atoms has the highest first ionization energy ?

(1*) Sc (2) Rb (3) Na (4) K

76. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are :

(1*) Four moles of NaOH and one mole of Br_2. (2) One mole of NaOH and one mole of Br_2

(3) Four moles of NaOH and two moles of Br_2 (4) Two moles of NaOH and two moles of Br_2

Sol. $R-\overset{\overset{O}{||}}{C}-NH_2 + 4NaOH + Br_2 \longrightarrow RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O$

77. Which of the following compounds is metallic and ferromagnetic ?

(1) MnO_2 (2) TiO_2 (3*) CrO_2 (4) VO_2

Sol. CrO_2 is metallic and ferromagnetic.

78. Which of the following statements about low density polythene is FALSE?

(1*) It is used in the manufacture of buckets, dust-bins etc.

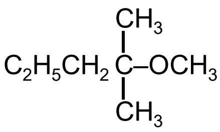
(2) Its synthesis requires high pressure.

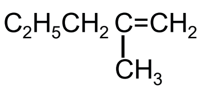
(3) It is a poor conductor of electricity.

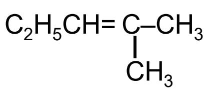
(4) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.

Sol. HDPE is used in making buckets and dustbins

79. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields :

(a) 

(b) 

(c) 

(1) (a) and (b) (2*) All of these (3) (a) and (c) (4) (c) only

Sol. via S_N1 and E_2 mechanisms

80. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by :

(1*) $\sqrt{2meV}$ (2) meV (3) 2meV (4) $\sqrt{meV}$

Sol. $\lambda = \frac{h}{mv}$

$= \frac{h}{\sqrt{2m(KE)}}$

$\therefore \frac{h}{\lambda} = \sqrt{2meV}$

81. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is:

(1) 759.0 (2) 7.6 (3) 76.0 (4*) 752.4

Sol. $n_{C_6H_{12}O_6} = 0.1$

$n_{H_2O} = \frac{178.2}{18} = 9.9$

$$\frac{p^0 - p}{p^0} \Rightarrow \frac{0.1}{10}$$

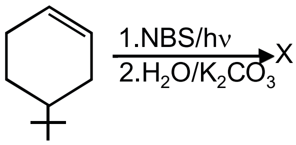
$$p^0 - p \Rightarrow \frac{0.1}{10} \times 760 = 7.6$$

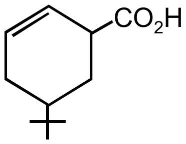
$p = p^0 - 7.6$

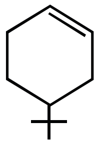
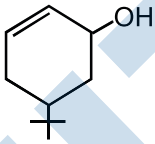
$= 760 - 7.6$

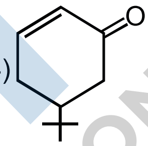
$= 725.4$ torr

82. The product of the reaction given below is:



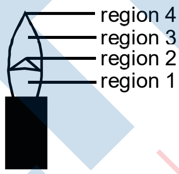
(1) 

(2) 
(3*) 

(4) 

Sol. Allylic bromination followed by hydrolysis.

83. The hottest region of Bunsen flame shown in the figure below is :



(1) region 4 (2) region 1 (3*) region 2 (4) region 3

Sol. Allylic bromination followed by hydrolysis.

84. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces

(1) NO_2 and N_2O (2*) N_2O and NO_2 (3) NO_2 and NO (4) NO and N_2O

Sol. $Zn + \underset{\text{conc.}}{4HNO_3} \longrightarrow Zn(NO_3)_2 + 2H_2O + + 2NO_2 \uparrow$

$4Zn + \underset{\text{dil.}}{10HNO_3} \longrightarrow 4Zn\ NO + N_2O \uparrow + + 5H_2O$

85. Which of the following is an anionic detergent?

(1) Glyceryl oleate (2) Sodium stearate

(3*) Sodium lauryl sulphate (4) Cetyltrimethyl ammonium bromide

Sol. Sodium $^{+}$(lauryl sulphate)$^{-}$ is an anionic detergent.

86. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate:

(1) $CH_3–CHCl–CH_2^+$ (2) $CH_3 –CH^+–CH_2 –OH$

(3*) $CH_3– CH^+ –CH_2 –Cl$ (4) $CH_3– CH(OH) – CH_2^+$

Sol. $CH_3 - CH = CH_2 + Cl^{\oplus} \longrightarrow CH_3 - \overset{\oplus}{C}H - CH_2Cl$ (Markovnikov's addition)

87. For a linear plot of log (x / m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(1) log (1/n) appears as the intercept

(2) Both k and 1/n appear in the slope term.

(3) 1/ n appears as the intercept.

(4*) Only 1/n appears as the slope

Sol. $\frac{x}{m} = kp^{\frac{1}{n}}$

$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

88. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(1*) Li_2O, Na_2O_2 and KO_2 (2) Li_2O, Na_2O and KO_2

(3) LiO_2, Na_2O_2 and K_2O (4) Li_2O_2, Na_2O_2 and KO_2

Sol. $\frac{x}{m} = kp^{\frac{1}{n}}$

$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$

89. The equilibrium constant at 298 K for a reaction A +B C + D is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be :

(1) 1.182 (2) 0.182 (3) 0.818 (4*) 1.818

Sol.

A +	B	C +	D
1	1	1	1
1–x	1–x	1+x	1+x

Given K_{eq} = 100

$\frac{(1+x)^2}{(1-x)^2} = 100$

$\frac{1+x}{1-x} = 10$

1 + x = 10 – 10x

11x = 9

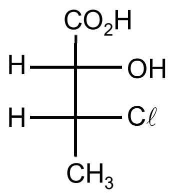
$x = \frac{9}{11}$

$[D] = 1 + \frac{9}{11}$

=1 + 0.818

= 1.818

90. The absolute configuration of



is :

(1) (2R,3R) (2) (2R, 3S) (3*) (2S,3R) (4) (2S, 3S)

Sol.